

## Rational inflation expectations in ADAM

### Resumé:

*We apply rational expectations for expected inflation in the user cost relations. The model properties under rational expectations are compared with standard ADAM with a multiplier analysis. The short term response in investment and output is larger and the crowding out time is shorter under rational expectations.*

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DSI

Nøgleord: Rational expectations, inflation

*Modelgruppepapirer er interne arbejdsrapporter. De konklusioner, der drages i papirerne, er ikke endelige og kan være ændret inden opstillingen af nye modelversioner. Det henstilles derfor, at der kun citeres fra modelgruppepapirerne efter aftale med Danmarks Statistik.*

## 1. Introduction

The paper, DSI15111, examined rational expectations (RE) in the housing sub-model for expected inflation, which directly enters the user cost relation. It was necessary to consider a small sub-model due to technical problems in solving a large model with RE in Pcim. This exercise can now be repeated for the whole model as Gekko is equipped with the Fair-Taylor method for solving models with leaded variables. This has been tested in SOA23114, where a rational expectation is considered in the wage equation, macro-consumption equation and housing model. A more detailed discussion of expectation formation can also be found in DKN161213. In the present paper, we consider rational expectations for all expected inflations in the user cost relations.

## 2. Inflation Expectations

The user cost relation for machinery and buildings includes a term for expected inflation,  $\pi^e$ , which is formulated as adaptive expectation:

$$\pi_t^e = \theta \cdot \left( \frac{pi_t}{pi_{t-1}} - 1 \right) + (1 - \theta) \cdot \pi_{t-1}^e \quad (1)$$

Where  $pi_t$  is investment price. Current expectations of future inflation depend on past expectations and an error adjustment term  $\theta$  in which current expectations are adjusted according to current actual inflation. The model property will depend on the values of  $\theta$  and the lag structures in (1), these are explored in TMK12901. Currently, all inflation expectations are exogenous in ADAM to ensure stability in the model.

In this paper we replace (1) with rational expectation of the form:

$$\pi_t^e = \frac{1}{3} \cdot \left( \frac{pi_{t+1}}{pi_t} + \frac{pi_{t+2}}{pi_{t+1}} + \frac{pi_{t+3}}{pi_{t+2}} \right) - 1 \quad (2)$$

The theory of rational expectation assumes that expectations are model-consistent and agents do not make systematic errors in their forecast. Equation (2) postulates that on average agent's inflation forecast will be correct and is equal to the average inflation in three future time periods. Using three leaded terms is necessary for the model to converge, see also SOA23114. We apply (2) to a total of 14 inflation expectation equations in ADAM, see appendix for a complete list.

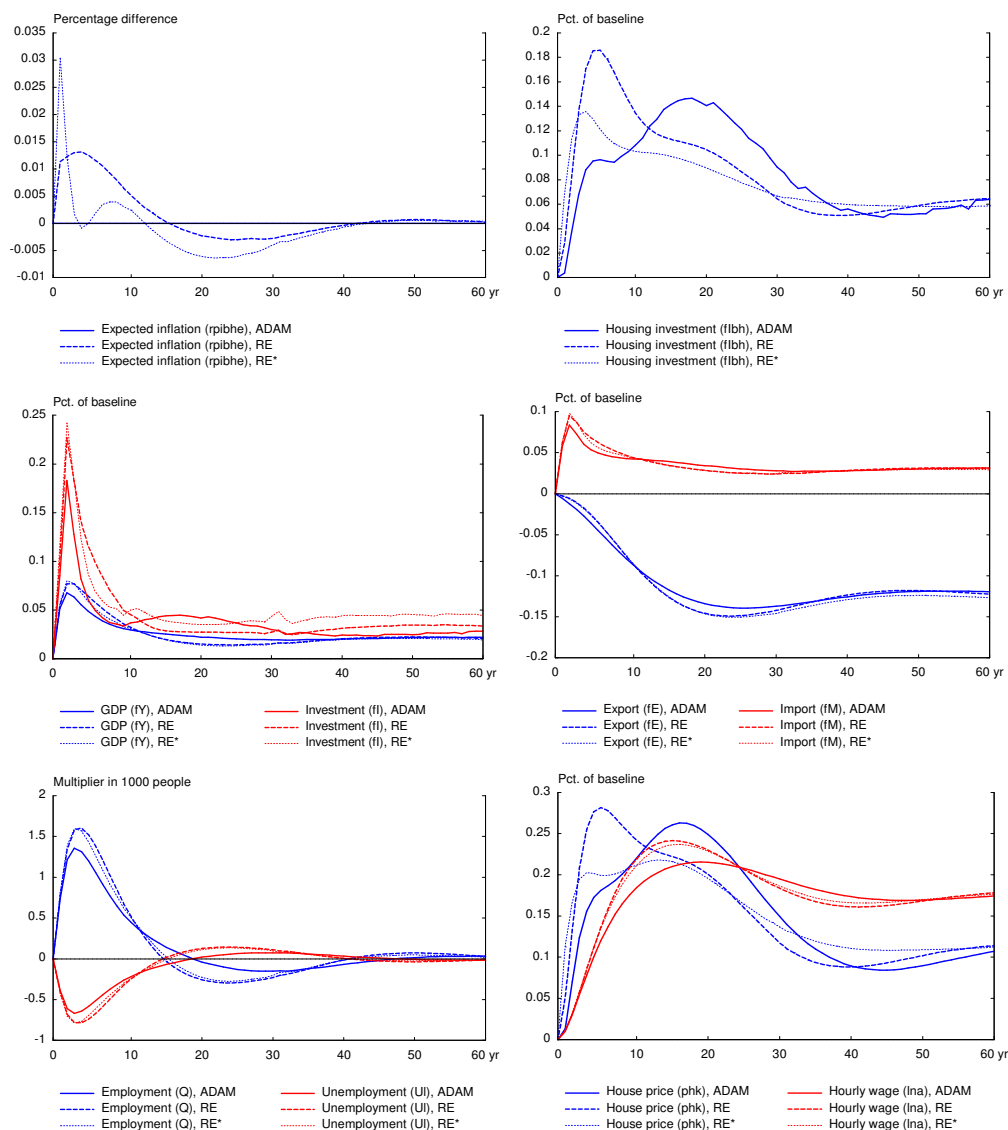
## 3. Multiplier Analysis

Here we compare the model properties under rational inflation expectation with standard ADAM through a multiplier analysis. We consider the following shocks in the model version july-13: i) Demand side shock – an increase in public purchase of goods and services; ii) Supply side shock – an increase in labour supply; iii) a shock in interest rates – a permanent fall in interest rates.

**i) An increase in public purchase of goods and services.**

The public spending is permanently increased by 1 billion kroner. Figure 1 shows the effect on selected variables – the label ‘ADAM’ is for standard july-13 model version, and ‘RE’ and ‘RE\*’ are for july-13 model with rational inflation expectation, the difference is that in the former investment price  $pi_{bh}$  and in the latter house price  $phk$  is used for expected inflation in the housing model.

**Figure 1. The effect of a permanent increase in public spending**



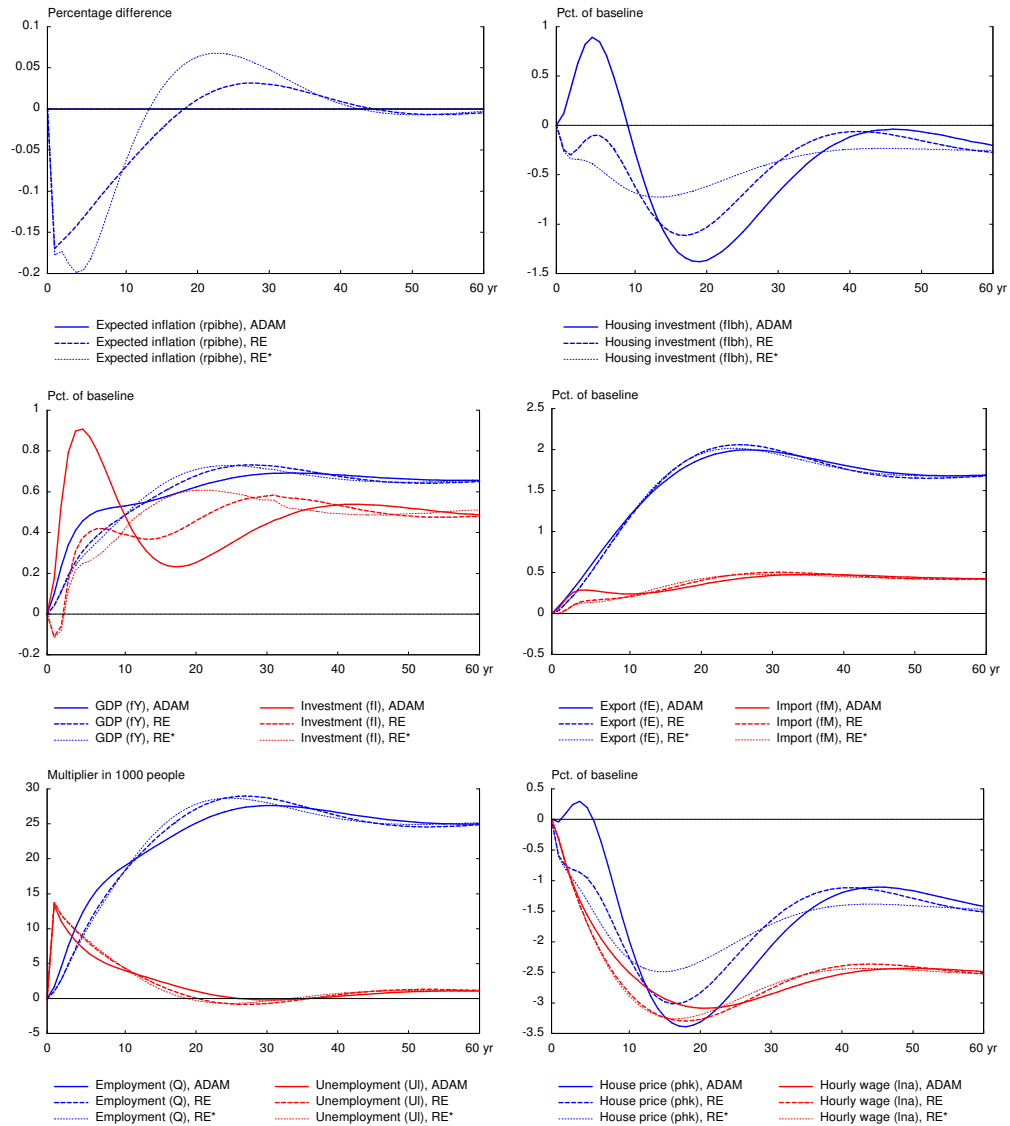
The higher public consumption expands total demand and both domestic production and imports increase relative to the baseline. The higher domestic economic activity raises employment. The lower unemployment eventually pushes wages and thus prices upward and exports decline and imports increase relative to the baseline. Eventually, the effect on employment returns to the baseline.

Under rational expectation, the initial response in investments is higher because agents know prices will be higher in the future and expected inflation increases already today and reduces user cost. The lower user cost expands investment. The strong initial response in investment is also reflected on total output and imports. The response of house price is also stronger. Agents know in advance that an increase in public spending will raise house prices and as they start buying houses prices increase. Overall, RE reduces the crowding out time from 20 in standard ADAM to 16 in RE and 17 in RE\*. Basing inflation expectation on house price (RE\*) prolongs the return of house price to the baseline, and slows down the whole process.

**ii) An increase in labour supply**

Labour force is permanently increased by approximately 1 percent of the total employment, which is achieved by reducing the number of people outside the labour force not receiving transfers with 27000.

**Figure 2. The effect of a permanent increase in labour supply**



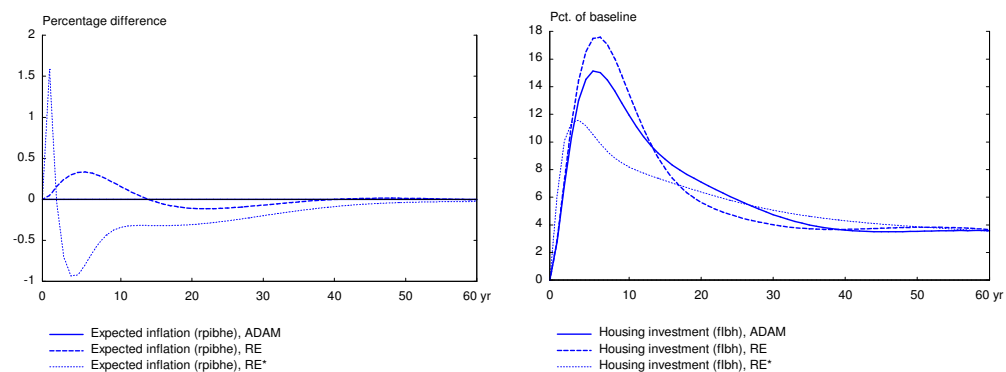
The increased labour supply cannot be soaked up automatically and as a result unemployment increases in the short run. The higher unemployment reduces the growth of wages and price. Production and exports expand and gradually pull the extra labour force into employment. Employment increase until the rate of unemployment is back to the baseline.

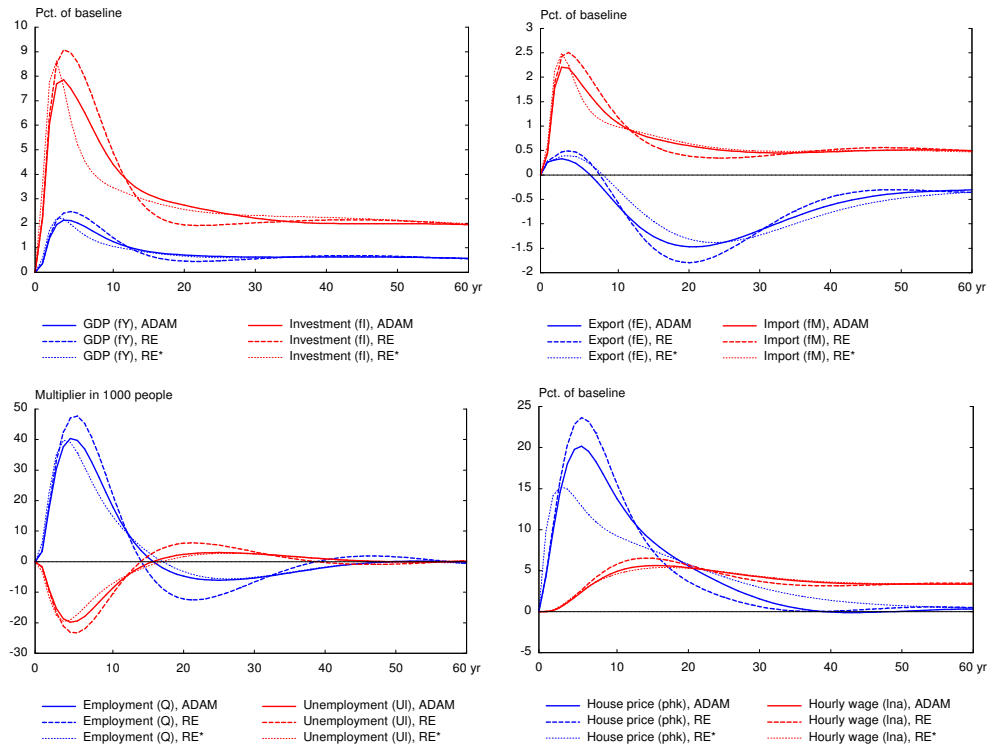
Unlike the demand side shock, the initial impact on investment under rational expectation is negative, due to largely the negative impact on housing investment. As agents expect prices to fall, user cost increases and investment will be postponed for future periods. The initial impact on investment under exogenous expectation is positive and significantly higher. The initial impact on employment is also higher with exogenous expectation. The role of RE in speeding up short term impacts is counter-productive in supply side shocks, where the first round impact on inflation is negative, but RE does formally reduce the crowding out time from 28 to 22 years. The long term multipliers under RE are no different from standard ADAM, this property is general.

### iii) A permanent fall in interest rates

Both the domestic and foreign interest rates are permanently reduced by 1 percentage point, from 3.5 to 2.5 percent. Investments expand as the cost of capital falls. Investment in housing and house price also increase. The rational expectation creates additional positive impact on investments as agents expect prices to increase in the future, which makes user costs fall today and expand investments. As in the above two cases the long-term impact on unemployment is zero due to the wage-driven crowding out process. Here too, the crowding out time is reduced in RE but not in RE\*. The drop in interest rates has little long-term impact on house price and the RE\* mechanism prolongs the return of house price to its baseline. The initial impact on house price is larger and quickly peaks and starts a downward adjustment process than that of the investment deflator. Hence, when agents base their inflation expectation on house price they revise down their inflation expectation quickly and the impact on expected inflation is already negative in the second year. This will dampen the short-term impact on housing investment and house price.

**Figure 3. The effect of a permanent fall in interest rates**





#### 4. Summary

The paper demonstrated the role of rational inflation expectation in ADAM. It is now possible to solve ADAM in Gekko with leaded variables, however, there is still some difficulty – more than one lead is required for the model to converge. To reduce the impact of potential problems it may be an idea to supplement Gekko’s Fair-Taylor algorithm with a newton procedure.<sup>1</sup> Using more than one leaded values dampens potential fluctuation, but it can also elongate the crowding out time. Under RE the initial effect on investment and output is pronounced and the crowding out time can also be shortened.

<sup>1</sup> See e.g. Brayton (2011). Two Practical Algorithms for Solving Rational Expectation Models. working paper, Federal Reserve Board.

## Appendix

The rational inflation expectation equations are the following

- (1)  $R_{pcbe} = (1/3) * (pm7b(+1)/pm7b + pm7b(+2)/pm7b(+1) + pm7b(+3)/pm7b(+2)) - 1$
- (2)  $R_{pibhe} = (1/3) * (pibh(+1)/pibh + pibh(+2)/pibh(+1) + pibh(+3)/pibh(+2)) - 1$
- (2\*)  $R_{pibhe} = (1/3) * (phk(+1)/phk + phk(+2)/phk(+1) + phk(+3)/phk(+2)) - 1$
- (3)  $R_{pibpe} = (1/3) * (pibp(+1)/pibp + pibp(+2)/pibp(+1) + pibp(+3)/pibp(+2)) - 1$
- (4)  $R_{pimae} = (1/3) * (pima(+1)/pima + pima(+2)/pima(+1) + pima(+3)/pima(+2)) - 1$
- (5)  $R_{pimbe} = (1/3) * (pimb(+1)/pimb + pimb(+2)/pimb(+1) + pimb(+3)/pimb(+2)) - 1$
- (6)  $R_{pimee} = (1/3) * (pime(+1)/pime + pime(+2)/pime(+1) + pime(+3)/pime(+2)) - 1$
- (7)  $R_{pimnge} = (1/3) * (pimng(+1)/pimng + pimng(+2)/pimng(+1) + pimng(+3)/pimng(+2)) - 1$
- (8)  $R_{pimnee} = (1/3) * (pimne(+1)/pimne + pimne(+2)/pimne(+1) + pimne(+3)/pimne(+2)) - 1$
- (9)  $R_{pimnfe} = (1/3) * (pimnf(+1)/pimnf + pimnf(+2)/pimnf(+1) + pimnf(+3)/pimnf(+2)) - 1$
- (10)  $R_{pimnze} = (1/3) * (pimnz(+1)/pimnz + pimnz(+2)/pimnz(+1) + pimnz(+3)/pimnz(+2)) - 1$
- (11)  $R_{pimqze} = (1/3) * (pimqz(+1)/pimqz + pimqz(+2)/pimqz(+1) + pimqz(+3)/pimqz(+2)) - 1$
- (12)  $R_{pimqse} = (1/3) * (pimqs(+1)/pimqs + pimqs(+2)/pimqs(+1) + pimqs(+3)/pimqs(+2)) - 1$
- (13)  $R_{pimqfe} = (1/3) * (pimqf(+1)/pimqf + pimqf(+2)/pimqf(+1) + pimqf(+3)/pimqf(+2)) - 1$
- (14)  $R_{pimoe} = (1/3) * (pimo(+1)/pimo + pimo(+2)/pimo(+1) + pimo(+3)/pimo(+2)) - 1$