## Definition

- A time series
  "Measures the *same phenomenon*
  at *equal intervals* of time"

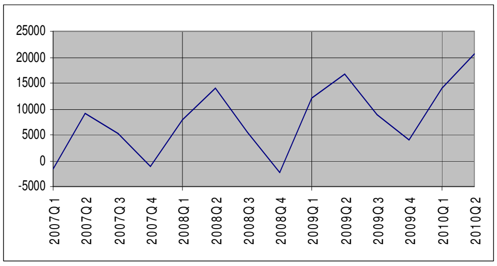


## Time series – Data

| Date | Value |
|---|---|
| 2007Q1 | 522 |
| 2007Q2 | 11622 |
| 2007Q3 | 2323 |
| 2007Q4 | -5105 |
| 2008Q1 | 6804 |
| 2008Q2 | 14044 |
| 2008Q3 | 6263 |
| 2008Q4 | 1229 |
| 2009Q1 | 8284 |
| 2009Q2 | 16701 |
| 2009Q3 | 13874 |
| 2009Q4 | 3792 |
| 2010Q1 | 14232 |
| 2010Q2 | 24967 |



## Time series – Graph



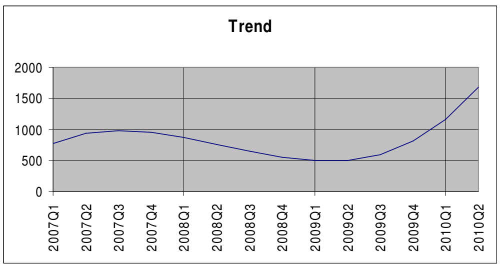



## Components of time series

- **Trend**: underlying long-term movement
- **Cycle**: medium-term cyclical movements about the trend
- **Seasonal (S)**: factors that occur one or more times per year. Stable in size and direction from year to year.
- **Irregular (I)**: residual after other components have been removed. Should exhibit no pattern.
- We combine the trend and the cycle to form trend-cycle (C), but refer to this as the "trend".

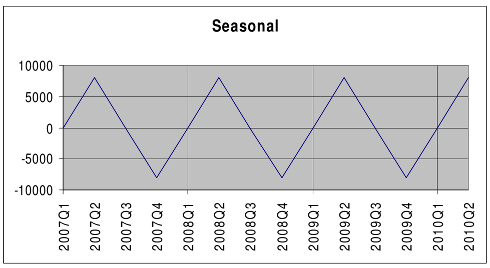


## Time series – Trend

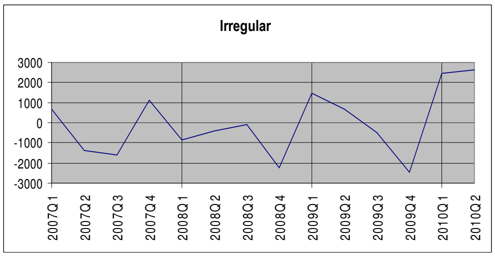





## Time series – Seasonal



## Time series – Irregular

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## Decomposition models

The components are unobserved and confounded. They can only be separately identified by making assumptions about their form.

Two models

- Additive: $Y_t = C_t + S_t + I_t$
- Multiplicative: $Y_t = C_t \times S_t \times I_t$

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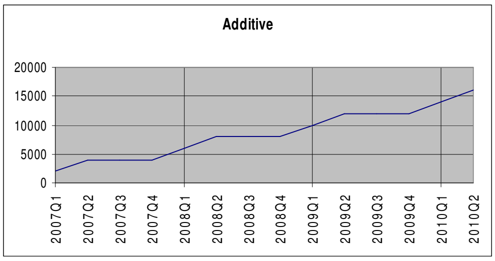
## Time series – Additive

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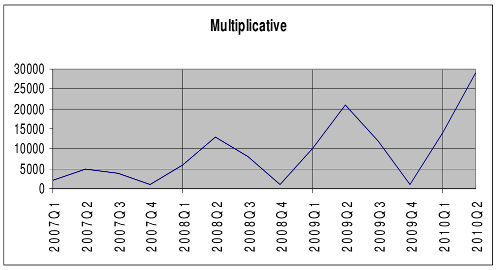
## Time series – Multiplicative

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## Seasonal adjustment – Estimation

- The aim of seasonal adjustment is to estimate the seasonal component of the time series and remove it.
- Additive: $SA(Y_t) = Y_t - S_t = C_t + I_t$
- Multiplicative: $SA(Y_t) = Y_t / S_t = C_t \times I_t$

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## Seasonal adjustment approach

- The X-11 approach to estimation is sequential, based on Macaulay (1931)
  - estimate easiest component first
  - remove easiest component leaving others
  - estimate others
- X-11 is a product of the US Census Bureau
  - 1954: X-0, X-1, … (Julius Shiskin)
  - 1967: X-11 Variant of the Census Method II Seasonal Adjustment Program (Shiskin, Young, Musgrave)

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## The X-11 method

- X-11 uses a system of filtering to estimate the different components
- It runs through a cycle of “estimation and improvement” three times.
- The primary filters used moving averages (MA), which are an example of “linear filters”.



## Moving averages

If $\{y_t\}$ is a time series of values

$$M(y_t) = \sum_{j=-n_1}^{n_2} w_{t+j} y_{t+j}$$

where $M(y_t)$ is an MA of order $(n_1 + n_2 + 1) < N$ successive terms and often:

$$\sum_{j=-n_1}^{n_2} w_{t+j} = 1$$

If $n_1 = n_2$ then $M(y_t)$ is said to be "centred"

If $w_{t-j} = w_{t+j}$ for all $j$ then $M(y_t)$ is said to be "symmetric"



## Centring moving averages

An MA with an order which is odd can be centred on its middle value.

A simple 3-term MA:

$$M(y_t) = \frac{1}{3}(y_{t-1} + y_t + y_{t+1})$$

| Period | Values | 3-term MA |
|---|---|---|
| 1 | 100 | |
| 2 | 103 | 102.3 |
| 3 | 104 | 104.3 |
| 4 | 106 | 104.0 |
| 5 | 102 | 104.3 |
| 6 | 105 | 105.3 |
| 7 | 109 | |



## Centring moving averages

For an MAwith an even order, the centre of the MA falls between values.

For example, a simple 4-term MA

$$\frac{1}{4}(y_{t-1} + y_t + y_{t+1} + y_{t+2})$$

The middle of the MA is between the second and third terms.

The solution: take a 2-term MA of the original 4-term MA.

| Period | Value | 4 – term MA | 2X4 MA |
|---|---|---|---|
| 1 | 100 | | |
| | | | |
| 2 | 103 | | |
| | | 103.25 | |
| 3 | 104 | | 103.5 |
| | | 103.75 | |
| 4 | 106 | | 104.0 |
| | | 104.25 | |
| 5 | 102 | | |
| | | | |
| 6 | 105 | | |



## Centring moving averages

This is known as a 2x4 MA.

More generally, we can express such "composite" MAs as n x m.

For the 2 x 4 composite MA:

$$M(y_t) = \frac{1}{2}\left[\frac{1}{4}(y_{t-2} + y_{t-1} + y_t + y_{t+1}) + \frac{1}{4}(y_{t-1} + y_t + y_{t+1} + y_{t+2})\right]$$

$$= \frac{1}{8}y_{t-2} + \frac{2}{8}y_{t-1} + \frac{2}{8}y_t + \frac{2}{8}y_{t+1} + \frac{1}{8}y_{t+2}$$

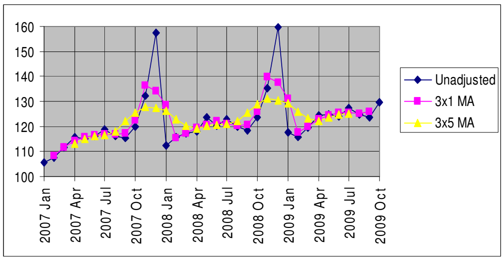
The weights are written $\frac{1}{8}[1, 2, 2, 2, 1]$.



## Seasonal adjustment – MA problem

- The X-11 method uses moving averages (MAs) for seasonal adjustment
- The problem with MAs is …

## Example: MAs

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## Seasonal adjustment – MA problem

- The X-11 method uses moving averages (MAs) for seasonal adjustment
- The problem with MAs is …
- … end points and outliers
- The solution for X-11 was asymmetric moving averages
- However, research in the 1970’s proved ARIMA forecasting - to enable symmetric moving averages - is better (ie lower revisions)

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## Seasonal adjustment – X-11

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  - 1975: Box-Jenkins ARIMA models used to develop X-11-ARIMA (Dagum, Statistics Canada)

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  - 1998: RegARIMA options used in the development of X-12-ARIMA (Findley, USCB)

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  - 1998: RegARIMA options used in the development of X-12-ARIMA (Findley, USCB)
  - 2012: Added SEATS decomposition and renamed X-13ARIMA-SEATS

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## Analysing a time series using X-12 ARIMA

1. Choose a decomposition
2. Fit a regARIMA model to clean and forecast the series
3. Seasonally adjust with X-11 method
4. Use diagnostics to assess your adjustment

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## The X-11 algorithm

- The X-11 approach contains four cycles (labelled A-D).
- The A-cycle consists of prior adjustment (cleaning the data).
- While the B- to D-cycles represent the iterative part of the algorithm. Each iteration leads to better estimates of the components

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## The B-cycle (assuming monthly data and an additive model)

1. Preliminary estimation of C using a 2x12 MA:

$$C_t^{(1)} = M_{2X12}(y_t)$$

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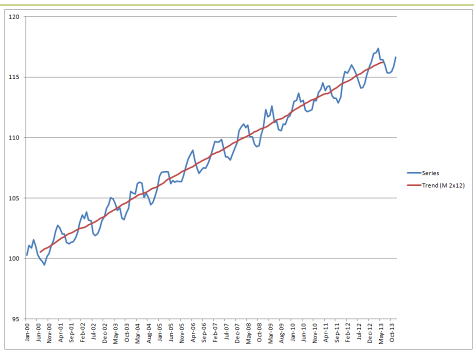
## Preliminary estimation of trend using a 2x12 MA

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## The B-cycle (assuming monthly data and an additive model)

1. Preliminary estimation of C using a 2x12 MA:

$$C_t^{(1)} = M_{2X12}(y_t)$$

2. Preliminary estimation of S+I component

$$(S_t + I_t)^{(1)} = y_t - C_t^{(1)}$$

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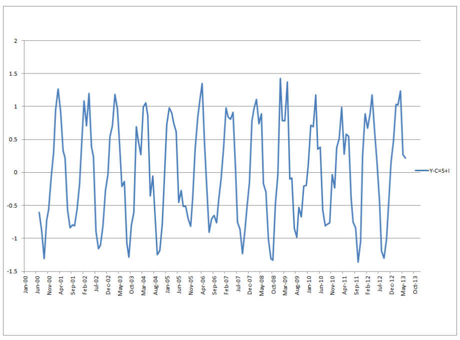
## Preliminary estimation of S+I component

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## The B-cycle (assuming monthly data and an additive model)

1. Preliminary estimation of C using a 2x12 MA:

$$C_t^{(1)} = M_{2X12}(y_t)$$

2. Preliminary estimation of S+I component

$$(S_t + I_t)^{(1)} = y_t - C_t^{(1)}$$

3 Preliminary estimation of S using a 3x3 MA:

$$S_t^{(1)} = M_{3X3}\left((S_t + I_t)^{(1)}\right)$$

$$\tilde{S}_t^{(1)} = S_t^{(1)} - M_{2X12}\left(S_t^{(1)}\right)$$

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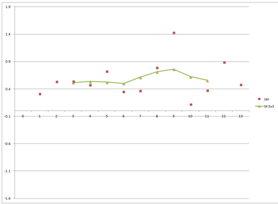
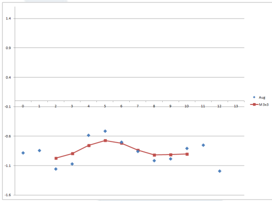
## Preliminary estimation of seasonal using a 3x3 MA

January



August





## The B-cycle (assuming monthly data and an additive model)

1. Preliminary estimation of C using a 2x12 MA:

$$C_t^{(1)} = M_{2X12}(y_t)$$

2. Preliminary estimation of S+I component

$$(S_t + I_t)^{(1)} = y_t - C_t^{(1)}$$

3 Preliminary estimation of S using a 3x3 MA:

$$S_t^{(1)} = M_{3X3}\left((S_t + I_t)^{(1)}\right)$$

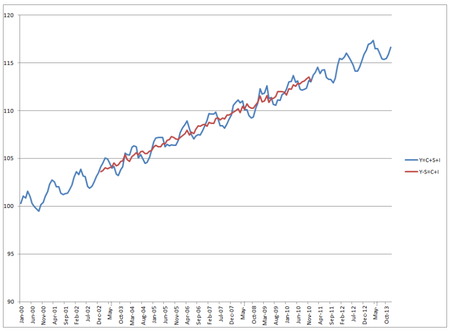
$$\tilde{S}_t^{(1)} = S_t^{(1)} - M_{2X12}\left(S_t^{(1)}\right)$$

4. Preliminary estimation of SA:

$$SA_t^{(1)} = (C_t + I_t)^{(1)} = y_t - \tilde{S}_t^{(1)}$$



## Preliminary estimation of SA





## Seasonal adjustment – X-11 algorithm

- After prior adjustment (‘A cycle’)
  - Apply Moving Average (MA) to B1 to estimate $C_1$
  - Remove $C_1$ from B1 to leave S and I (SoI)
  - Apply MA to SoI to estimate $S_1$
  - Remove $S_1$ from B1 to estimate CoI (seasonally adjusted)
  - Apply Henderson MA to CoI to estimate $C_2$
  - Remove $C_2$ from B1 to leave S and I (SoI)
  - Apply MA to SoI to estimate $S_2$
  - Remove $S_2$ from B1 to estimate CoI

Repeat in C and D cycles



## Henderson trend filters

- MA filters work well only when the series contains a linear trend.
- For this reason, the second estimates of trend is performed using a Henderson filter.
- The weights for a Henderson filter are computed to allow polynomial trends up to degree three to be exactly reproduced.



## Henderson trend filters

- Henderson weights can be constructed for any odd order from 5 upwards.
- Example weights:

$$5-term: \frac{1}{286}\{-21, 84, 160\}$$

$$13-term: \frac{1}{16796}\{-325, -468, 0, 1100, 2475, 3600, 4032\}$$

- The Henderson filters used in X-11 are:

  5-,7-term for quarterly series

  9-,13-,23-term for monthly series

## Selection of filter lengths

A longer filter produces a smoother trend (or seasonal factors) but is less responsive to changes in the trend (or seasonality).

The solution built into the X-11 is to use signal to noise ratios:

For example, the ratio for the trend in an additive decomposition is:

$$\frac{I}{C} = \frac{1}{n-1} \frac{\sum_{t=2}^{n} |I_t - I_{t-1}|}{\sum_{t=2}^{n} |C_t - C_{t-1}|}$$

A large ratio indicates that a long filter is needed.

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## Length of trend filter

| | Monthly series | Quarterly series |
|---|---|---|
| I/C < 1 | 9-term | 5-term |
| 1<I/C<3.5 | 13-term | 5-term |
| I/C > 3.5 | 23-term | 7-term |

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## Selection of seasonal filter lengths

- Again, as with the trend, the choice of length of filter is based on the noise to signal ratio.
- But here the ratio is between noise and seasonality.
- For, an additive series:

$$\frac{I}{S} = \frac{\sum_j \frac{n_j}{(n_j - 1)} \sum_{i=2}^{n} |I_{i,j} - I_{i-1,j}|}{\sum_j \frac{n_j}{(n_j - 1)} \sum_{i=2}^{n} |S_{i,j} - S_{i-1,j}|}$$

where $i$ = year, $j$ = month or quarter

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## Length of filter for the seasonal MA (sma)

| | |
|---|---|
| I/S<2.5 | sma=3x3 |
| 2.5<I/S<3.5 | Remove last year and recalculate |
| 3.5<I/S<5.5 | sma=3x3 |
| 5.5<I/S<6.5 | Remove last year and recalculate |
| I/S<6.5 | sma=3x3 |

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