Danmarks Statistik
MODELGRUPPEN

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Interest rates in ADAM

Resumé:

This paper summarizes the structure of interest rates in ADAM and suggests an alternative structure that is simple and pedagogically motivated. We suggest a simplified equation for bank loan rate and a simple additive relation for those interest rates determined as a simple markup over another interest rate.

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Key words: Interest rate, markup equation, additive equation

1. Introduction

This paper examines the structures and behavioural relations that determine interest rates in ADAM. We suggest an alternative flow of interest rates and behavioural equations. We conclude by comparing the alternative interest rate structures with a means of multiplier experiments.

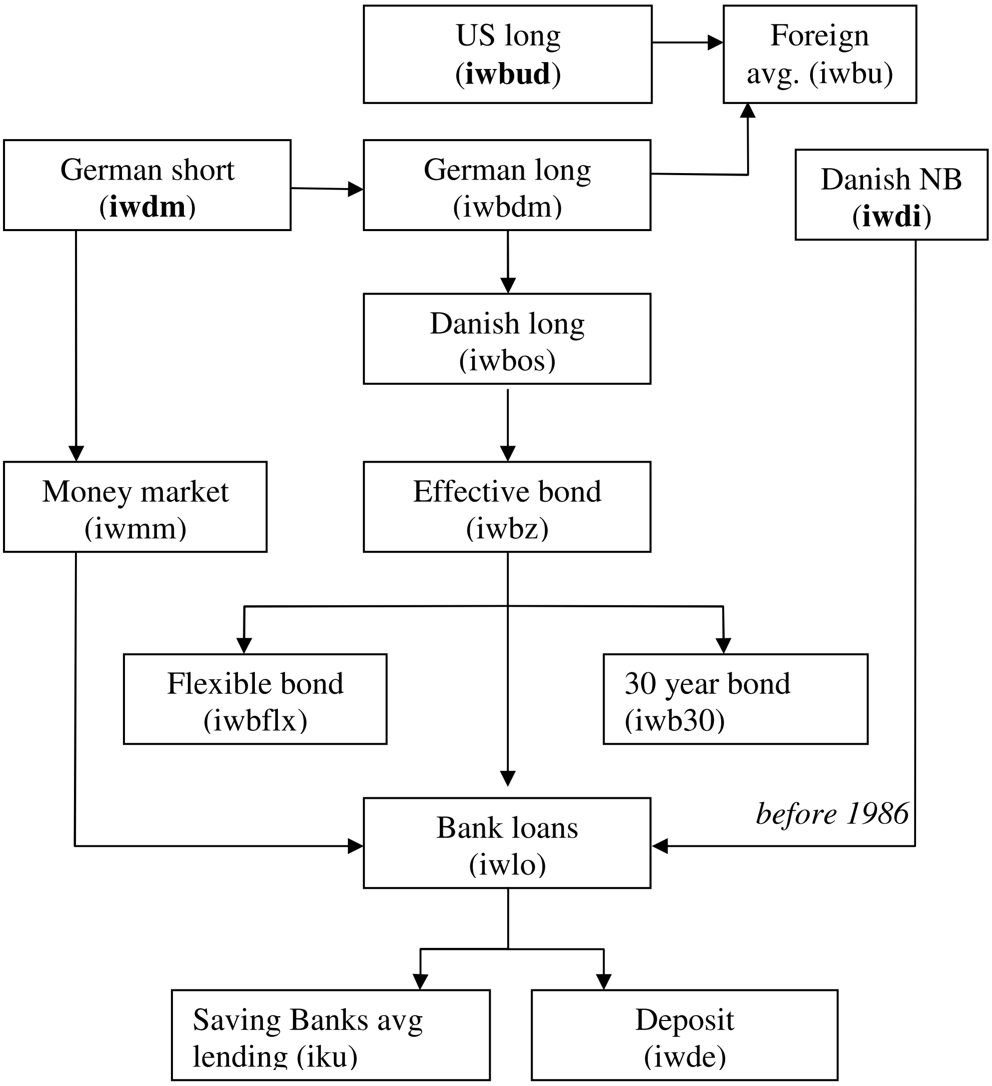
2. Interest rate structure

Interest rates in ADAM are determined based on foreign interest rates and Danish National Bank's discount rate. The charts below present the hierarchical flow of interest rate determinations. For ease of understanding we present the structures in two groups. The first flow diagram presents the interest rates determined as a simple linear function of the other interest rate(s).[1,2] The second diagram presents a group of interest rates which depend indirectly on the first group of interest rates and other variables in the model.

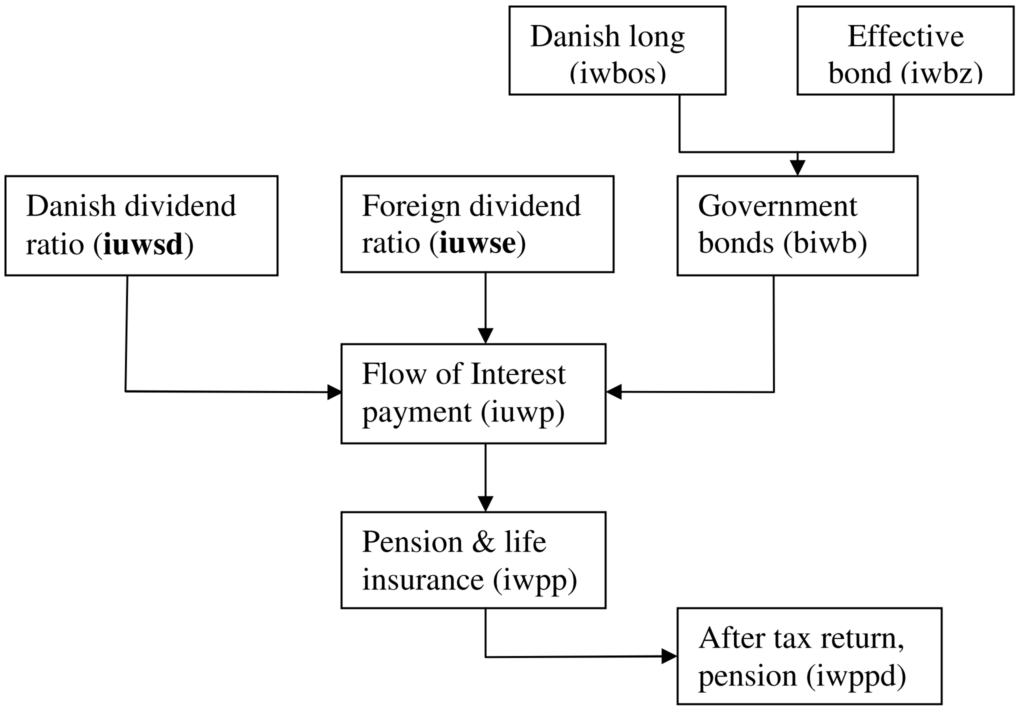
The process begins from the German long rate and discount rate and proceeds as indicated by the arrows. Most of the interest rates are determined as a simple mark-up (k-factor) over another interest rate and some are determined as an average or complicated functions of two or more interest rates, we will turn to this shortly.

[1] To keep the diagram simple, we have excluded the interest rates: iwbosu (determined from iwbos), iwbzsu (determined from iwbz), iwbzh (determined from iwbz) and iwbr (determined by iwbz, iwpp, & other variables).
[2]Exogenous interest rates are printed in bold in both diagrams.

Figure 1. Interest rate flow diagram (a)



Interest rate flow diagram (b)



iwbud	U.S. effective interest rate on longterm bonds
iwdm	German market interest rate (3 month deposit rate)
iwbdm	German effective interest rate on longterm bonds
iwbu	Weighted foreign interest rate
iwdi	Danish National bank interest rate
iwbos	Yields on 10 year government bond, annual averages
iwmm	Money market rate
iwbz	Effective bond rate
iwb30	Yields on 30 years bond
iwbflx	Flexabile bond rate
iwlo	Banks effective lending rate
iku	Banks and Savings Banks' average lending rate
iwde	Banks' effective deposit rate
iwbosu	Yields on 10 year government bond (bonds), end
iwbzsu	Interest rate of discounting of depreciation, adjusted for indexation
iwbr	Return rate for use in calculating the rate of real interest tax
biwb	Interest rates on government bonds
iuwse	Dividend ratio of foreign shares
iuwsd	Dividend ratio of Danish shares
iuwp	flow of interest payment including devidends
iwpp	return before tax, life insurance and pension
iwppd	After-tax return rate, pension, private sector excl. ld

3. The interest rate equations

The interest rates *iwlo* and *iwde* have an estimated relation in ADAM that accounts regime shifts in the interest rate determinations since 1986. We will return to these later. The remaining interest rates are determined as a simple markup over another interest rate, which can be given as:

$$i_n = k_n * i_m \qquad (1)$$

Where, i_m is the predetermined interest rate such as the German long rate (*iwbdm*), i_n is the endogenous interest rate such as the Danish long rate (*iwbos*), and k_n is a mark-up or k-factor.

One implication of (1) is that an '*x*' percentage point change in i_m does not imply an equivalent change in i_n provided k_n is different from one. This will hold if one instead considers percentage change in interest rates. Then the question is should one consider percentage point change or percentage change, we argue for the former. It is also natural to ask if a different direction, say for example the money market rate directly determining the flexible bond rate, is a better formulation.

4. An alternative interest rate structure and equation

Figure 2 presents an alternative interest rate structure. The basic change is that we do not take the trouble of relating the German long rate to the German short rate. We keep both of these interest rates exogenous and relate the Danish short rate to German short rate and Danish long rate to German long rate.

The next question is to derive an alternative formulation to (1) such that an 'x' percentage point change on the right hand side implies an equivalent change on the left hand side. This is described below, see also Sørensen and Whitta-Jacobsen (2005), chapter 23.

Consider the German and Danish interest rates on long term bonds. A unit of German currency invested on German bonds yields an amount (1 + *iwbdm*). Alternatively, the foreign investor can buy 1*E units of Danish currency (where E is nominal exchange rate, Danish Krone per unit of German D-mark) and invest it on Danish bonds. At the end of the period, the investor will receive E(1 + *iwbos*) in Danish currency. At the start of the period the investor expects this end of period return to be $E/E_{+1}^{e}(1+iwbos)$ units of German currency, where E_{+1}^{e} is the nominal exchange rate expected to prevail. Provided exchange rates are completely fixed, $E/E_{+1}^{e}=1$, perfect mobility of capital implies the arbitrage condition: $1+iwbdm=1+iwbos \Rightarrow iwbdm=iwbos$.

In reality, foreign and domestic interest rate are not equal, nor is the exchange rate credibly fixed. A German investor holding Danish bonds can experience a capital loss if the Danish Krone depreciates against the D-mark. A simple way of modelling this uncertainty is to attach probabilities for exchange rate fluctuations. Let q be the degree of depreciation with a probability p, expected return on Danish bonds will be equal to

$$(1-p)(1+iwbos)+p(1-q)(1+iwbos)$$

And from the arbitrage condition we have

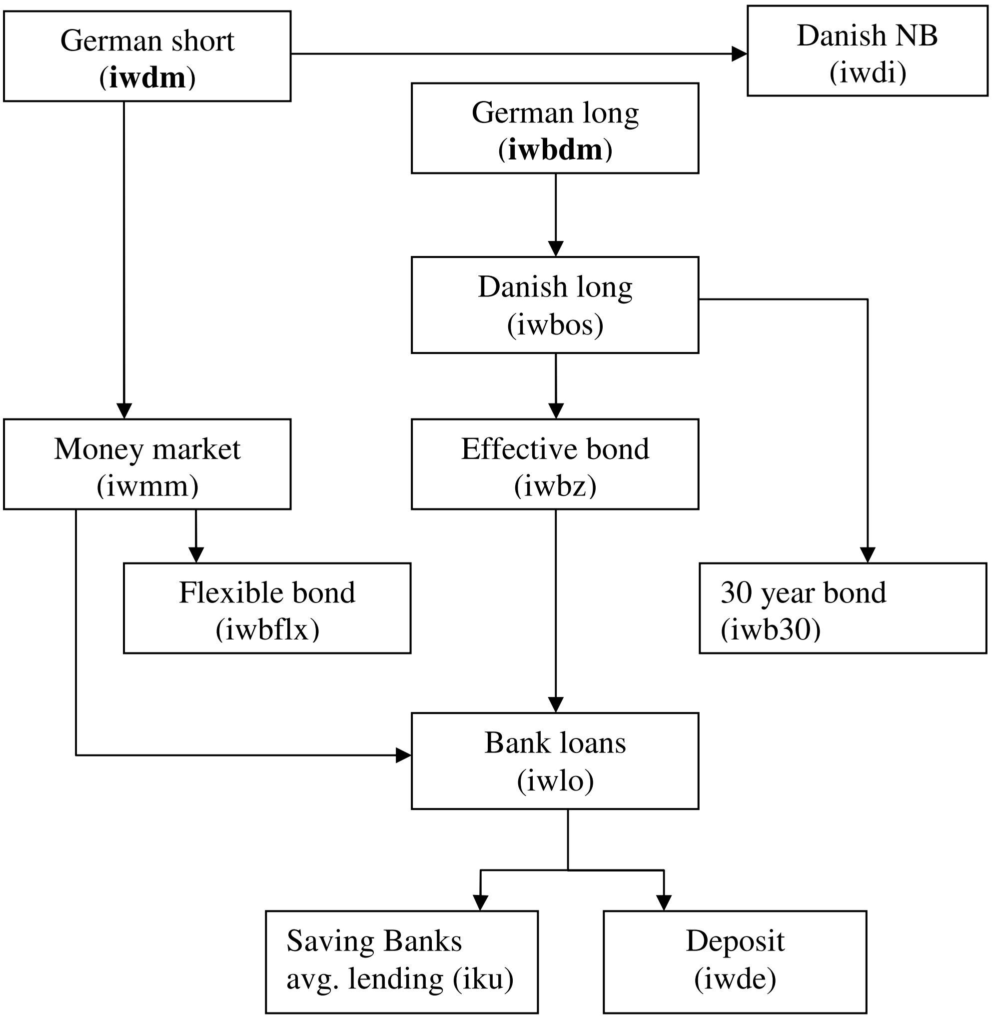
$$1+iwbdm=(1-p)(1+iwbos)+p(1-q)(1+iwbos) \qquad (2)$$

Note $\ln(1+x)\approx x, 0 \prec x \prec 1$, taking natural logarithm of (2) and rearranging, we have

$$iwbdm = iwbos - pq$$
$$iwbos = iwbdm + premium \qquad (3)$$

The additive construction (3) is pedagogically motivated and has the property that an '*x*' percentage point change in the German rate will correspond to an equivalent 'x' percentage point change in the Danish rate. Hence, we propose the interest rate relation (3) and the alternative structure of interest rates (figure 2). We also propose the additive structure (3) to *iwpp* in Figure 1b. We will now proceed to bank loans and deposit rates.

Figure 2. An alternative interest rate structure



5. Bank loans and deposit rates

The present equations for bank loan rate, *iwlo*, and deposit rate, *iwde*, are not easily understandable and do not facilitate straightforward interpretation of the equations. This is due to the different shift and blip dummies included to account for regime shifts after 1986 in the interest rate determination. If one considers only the period after 1986, there will be no need for the different deterministic terms. One case where the deterministic terms would be of importance is if one wants to make historical simulations. Taking these facts into consideration, we reformulate the equations for the two interest rates as follow.

$$\begin{aligned} iwlo_{old} = kiwlo * [& \gamma_1 * (1\text{-}Dwral) * iwbz + \gamma_2 * (1\text{-}Dwral) * iwmm \\ & + \gamma_3 * \mathrm{dw86} * iwmm \\ & + \varphi_1 * Dwral * iwdi + \varphi_2 * (1\text{-}Dwral\text{-}\mathrm{dw86}) * iwdi \\ & + \delta * Drml + k] \end{aligned} \tag{4}$$

$$\begin{aligned} iwlo = & (1 - D6686) * (\beta_0 + \beta_1 * iwbz + \beta_2 * iwmm) \\ & + D6686 * iwlo_{old} \end{aligned}$$

$$\begin{aligned} iwde_{old} = kiwde * [& \gamma_1 * Dwrad) * iwbz + \gamma_2 * (1\text{-}Dwrad) * iwlo \\ & + \gamma_3 * (1 - Dward) * iwmm \\ & + \varphi_1 * Dwrad * iwdi + \varphi_2 * Dward + k] \end{aligned} \tag{5}$$

$$\begin{aligned} iwde = & (1 - D6686) * (\delta_0 + \delta_1 * iwlo + \delta_2 * iwmm) \\ & + D6686 * iwde_{old} \end{aligned}$$

All variables are as defined in ADAM, *D6686* is 1 before 1986 and zero otherwise. For the period post 1986, the interest rate equations boil down to a simple and more intuitive relation. Further, if the hypotheses $\beta_1 + \beta_2 = 1$ and $\delta_1 + \delta_2 = 1$ can not be rejected, we will have a more intuitive structure, where a 1 percentage point change in the right hand side of the equation implies an equivalent 1 percentage point change in the dependent variable. Table 1 and 2 presents the estimation results.

Table 1. Estimation result, dependent variable *iwlo*

Variable	Parameter	(A)	(B)
iwbz	β_1	0.902 [0.155]	0.636 [*]
iwmm	β_2	0.261 [0.129]	0.364 [0.144]
Const.	β_0	0.004 [0.005]	0.017 [0.003]
		n = 1986-2007, LR=80.34, adj-R^2 = 0.957	n = 1986-2007, LR = 76.47, adj-R^2 = 0.956

Table 2. Estimation result, dependent variable *iwde*

Variable	Parameter	(A)	(B)
iwlo	δ_1	0.308 [0.048]	0.409 [*]
iwmm	δ_2	0.397 [0.049]	0.591 [0.200]
Const.	δ_0	-0.00536 [0.002]	-0.025 [0.005]
		n = 1986-2007, LR=103.34, adj-R^2 = 0.989	n = 1986-2007, LR = 71.41, adj-R^2 = 0.989

In both tables column (B) reports the estimation result with homogeneity restriction on the two slop coefficients. The homogeneity restriction with LR-test statistic of 7.74 = 2(80.84-76.47) and 63.87=2(103.34-71.41) is rejected with a significance level of 0.005 and 0.000, respectively. It should be noted that a test for homogeneity can not be performed on level regressions such as these ones due to unit roots, autocorrelation and other problems. Instead, one can check if the interest rates cointegrate homogeneously.

One can, however, argue for a homogenous restriction in the equation for *iwlo* by informally judging the unrestricted estimates. But this can not be the case for *iwde*, in part because it is strictly rejected and will also mean that if the weighted average value of *iwlo* and *iwmm* drop below 2.5%, the deposit rate will become negative, which is hardly true. Thus our preferred structures are a homogenous relationship for *iwlo* and unrestricted relation for *iwde*:

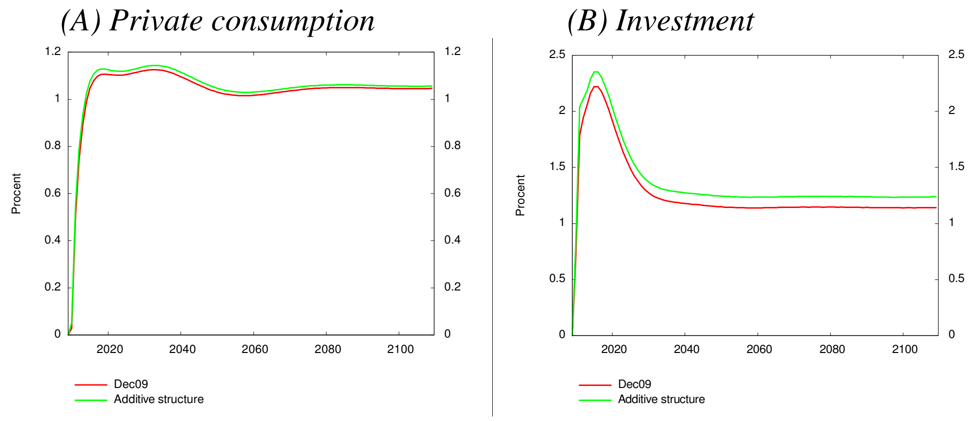
$$\begin{aligned} iwlo\ &= (1-D6686)*(0.0002+0.636*iwbz+0.364*iwmm) \\ &\quad +D6686*iwlo_{old} \end{aligned} \tag{6}$$

$$\begin{aligned} iwde &= (1-D6686)*(-0.005+0.308*iwlo+0.397*iwmm) \\ &\quad +D6686*iwde_{old} \end{aligned} \tag{7}$$

6. Multiplier experiment

Here we compare multiplier experiment results under the alternative interest rate formulations. We permanently reduce the exogenous interest rates by 0.005 point (*iwdm, iuwsd* and *iuwse* in Dec09; *iwdm, iwbdm, iuwsd* and *iuwse* in the alternative interest rate formulation).

Figure 3. Private consumption and investment, multiplier in %.



The fall in the foreign interest rate corresponds to a proportional fall in the domestic interest rates and gives rise to an expansionary effect on private consumption and investment. The results are very close under the alternative models, the reason is that the k-factors in Dec09 has been set to 1, which corresponds to an additive structure with zero risk premium. The small difference observed is due to the difference in the equation for bank loan rates (*iwlo*). The fall in *iwlo* is less than 0.005 (0.003 to be precise) in Dec09, whereas 0.005 under the alternative formulation.

7. Conclusion

In the end, an additive structure produces no different result from a markup formulation when the k-factors are set to 1. For a k-factor different from 1 the two will produce different results. The additive structure is pedagogically motivated and will have an upper hand during forcasting. We simplify the interest rate structure by directly relating the German long to the Danish long rate and German short to Danish short rate, and the other domestic interest rates follow the Danish short and long rates. Finally, we think a simple homogenious relation for bank loan rate is a better alternative than the old equation.